

Nucleon sigma term and in-medium quark condensate in the modified quark-meson coupling model*

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Abstract

We evaluate the nucleon sigma term and in-medium quark condensate in the modified quark-meson coupling model which features a density-dependent bag constant. We obtain a nucleon sigma term consistent with its empirical value, which requires a significant reduction of the bag constant in the nuclear medium similar to those found in the previous works. The resulting in-medium quark condensate at low densities agrees well with the model independent linear order result. At higher densities, the magnitude of the in-medium quark condensate tends to increase, indicating no tendency toward chiral symmetry restoration.

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The physics of nuclear matter and finite nuclei is governed by the underlying theory of strong interactions of quarks and gluons, quantum chromodynamics (QCD). In reality, however, QCD is intractable at the nuclear physics energy scales due to the nonperturbative features of the theory and a realistic account of nuclear phenomena based entirely on it is not yet possible. At this stage, the best one could do is to build models that incorporate the symmetries of QCD and/or quark-gluon degrees of freedom, and hence to motivate connections between this theory and the observed nuclear phenomena and established phenomenology. Such models are necessarily quite crude.

The quark-meson coupling model (QMC) [1] treats nucleons in nuclear medium as non-overlapping MIT bags interacting through the self-consistent exchange of mesons in the mean-field approximation. It provides a simple and attractive framework to incorporate the quark structure of the nucleon in the study of nuclear phenomena [1–6]. Recently, the QMC has been modified by introducing a density-dependent bag constant [7,8]. It was demonstrated that a significant reduction of the bag constant in the nuclear matter relative to its free space value can lead to large and canceling isoscalar Lorentz scalar and vector potentials and hence strong spin-orbit force for the nucleon in nuclear matter which are comparable to those suggested by relativistic nuclear phenomenology [9,10] and finite-density QCD sum rules [11]. Such a large reduction of the bag constant can also account for the EMC effect within the framework of dynamical scaling [12]. (For further development and other applications, see Refs. [13,14]).

In this paper, we evaluate the nucleon σ term and in-medium quark condensate in the modified quark-meson coupling model (MQMC) developed in Refs. [7,8]. The in-medium quark condensate plays an important role in studying hadron properties in nuclear medium [11] and has connections to many nuclear phenomena [15]. Its value at low densities is largely determined by the nucleon σ term which also has attendant consequences in nuclear astrophysics [16]. In Refs. [3,4] both the nucleon σ term and the in-medium quark condensate have been investigated in the QMC. There, it was found that the result for the nucleon σ term is much smaller than its empirical value extracted from dispersion analysis of isospin even pion-nucleon scattering [17], and the prediction for the in-medium quark condensate based on this small σ term is quite different from the model independent linear order result [18–20]. It is thus of interest to examine how the large reduction of the bag constant in the nuclear matter suggested in the MQMC will affect the nucleon σ term and in-medium quark condensate.

We find that when the bag constant is significantly reduced in the nuclear matter, e.g. $B/B_0 \sim 35 - 40\%$ at the nuclear matter saturation density, the empirical value for the nucleon σ term can be recovered and the resulting in-medium quark condensate at low densities is in good agreement with the model independent linear order result. At high densities, the magnitude of the in-medium quark condensate tends to increase, indicating no tendency of chiral symmetry restoration; this behavior has also been seen in other models [21–24]. Such a large reduction of the bag constant, as shown in previous works [7,8,12], is consistent with that required to recover large and canceling Lorentz scalar and vector potentials for the nucleon in the nuclear matter and to account for the EMC effect within the dynamical rescaling framework.

The details of the MQMC have been given in [7,8]. The two models for the in-medium bag constant are: scaling model and direct coupling model. The scaling model relates the in-medium bag constant to the in-medium nucleon mass :

$$\frac{B}{B_0} = \left[\frac{M_N^*}{M_N} \right]^\kappa, \quad (1)$$

where κ is a real positive parameter and $\kappa = 0$ corresponds to the usual QMC model. The effect of this modification is summarized into a factor $C(\bar{\sigma})$ that appears in the self-consistency condition for the σ field [7,8],

$$C(\bar{\sigma}) = \frac{E_{\text{bag}}}{M_N^*} \left[\left(1 - \frac{\Omega_q}{E_{\text{bag}} R} \right) S(\bar{\sigma}) + \frac{m_q^*}{E_{\text{bag}}} \right] \left[1 - \kappa \frac{E_{\text{bag}}}{M_N^{*2}} \frac{4}{3} \pi R^3 B \right]^{-1}, \quad (2)$$

where $m_q^* = m_q - g_\sigma^q \bar{\sigma}$, and the explicit expressions for M_N^* , E_{bag} , Ω_q , and $S(\bar{\sigma})$ can be found in Refs. [7,8]. The expression for $C(\bar{\sigma})$ in the usual QMC model is obtained from Eq. (2) with $\kappa = 0$. The direct coupling model features a direct coupling between the bag constant and the scalar mean field

$$\frac{B}{B_0} = \left[1 - g_\sigma^B \frac{4}{\delta} \frac{\bar{\sigma}}{M_N} \right]^\delta, \quad (3)$$

where g_σ^B and δ are real positive parameters and the introduction of M_N is based on the consideration of dimension. (The case $\delta = 1$ was also considered by Blunden and Miller [6].) Note that g_σ^B differs from the quark-meson coupling g_σ^q (or $g_\sigma \equiv 3g_\sigma^q$). When $g_\sigma^B = 0$, the usual QMC model is recovered. The factor $C(\bar{\sigma})$, in this case, is given by

$$C(\bar{\sigma}) = \frac{E_{\text{bag}}}{M_N^*} \left[\left(1 - \frac{\Omega_q}{E_{\text{bag}} R} \right) S(\bar{\sigma}) + \frac{m_q^*}{E_{\text{bag}}} \right] + \left(\frac{g_\sigma^B}{g_\sigma} \right) \frac{E_{\text{bag}}}{M_N^*} \frac{16}{3} \pi R^3 \frac{B}{M_N} \left[1 - \frac{4}{\delta} \frac{g_\sigma^B \bar{\sigma}}{M_N} \right]^{-1}. \quad (4)$$

The nucleon σ term can be expressed as

$$\sigma_N = m_q \frac{dM_N}{dm_q} = 2m_q \langle N | \bar{q}q | N \rangle. \quad (5)$$

Here we neglect isospin breaking and use $m_u = m_d \equiv m_q = \frac{1}{2}(m_u + m_d)$. Therefore, there are two ways to evaluate σ_N in QCD. One is to take the derivative of M_N with respect to m_q . The other is to calculate the nucleon's scalar charge $\langle N | \bar{q}q | N \rangle$, which can be carried out by adding a term $S(\bar{u}u + \bar{d}d)$ (with S a constant) to the QCD Hamiltonian and then extracting the response of the nucleon mass to the external field S , $dM_N(S)/dS|_{S \rightarrow 0} = \langle N | \bar{u}u + \bar{d}d | N \rangle$. The QMC and MQMC provide descriptions of how the nucleon mass responds to a constant scalar field. Treating $g_\sigma^q \bar{\sigma}$ as a constant external field, one can show

$$\sigma_N = m_q \frac{dM_N^*(S = -g_\sigma^q \bar{\sigma})}{dS} \Big|_{S \rightarrow 0} = 3m_q C(\bar{\sigma} = 0). \quad (6)$$

where $C(\bar{\sigma} = 0)$ is related to the response of the nucleon mass to the scalar field at $\sigma = 0$. The explicit expressions for $C(\bar{\sigma})$ are in the Eqs. (2) and (4).

We follow Refs. [3,4] and take a value of 10 MeV for m_q . We expect this value to be reasonable at the scale where the MIT bag model is useful. Note that $C(0)$ also depends on m_q . Here we use $m_q = 0$ in evaluating $C(0)$. It has been found in previous studies that inclusion of a small finite m_q in evaluating $C(0)$ only leads to a negligible refinement to the $C(0)$ value. Figure 1 shows the resulting σ_N from the scaling model as a function of κ for $R_0 = 0.6$ fm. When $\kappa = 0$, corresponding to the usual QMC, $\sigma_N \simeq 12$ MeV which is almost a factor of four smaller than the empirical value of 45 MeV [17]. As κ increases, σ_N increases slowly at small κ values and grows rapidly at large κ values. For $\kappa \simeq 2.88$, we find $\sigma_N \simeq 45$ MeV. We also find that the result is largely independent of R_0 in the range $0.6 \text{ fm} \leq R_0 \leq 1.0 \text{ fm}$. The result from the direct coupling model is illustrated in Fig. 2, where σ_N is plotted as a function of g_σ^B/g_σ with $R_0 = 0.6$ fm. We see that the dependence of σ_N on g_σ^B/g_σ is linear. The case $g_\sigma^B/g_\sigma = 0$ corresponds to the usual QMC. When $g_\sigma^B/g_\sigma \simeq 1.1$, the empirical value of σ_N can be reproduced. Our results for κ in the scaling model and g_σ^B/g_σ in the direct coupling model in order to obtain the empirical value of σ_N term depend slightly on the choice we make for m_q in the vicinity of $m_q \sim 10 \text{ MeV}$.

Therefore, it is necessary to have $\kappa > 0$ and $g_\sigma^B/g_\sigma > 0$ or a *reduction* of the bag constant in the nuclear medium in order to recover the empirical value of σ_N from the small value predicted by the usual QMC. If one had assumed the opposite behavior, the resulting σ_N would be even smaller. This solidifies the MQMC model. It is the coupling of the bag constant to the scalar field featured in the MQMC that describes how the bag constant responds to the external field and provides a new source of contribution to the nucleon's scalar charge and hence the nucleon σ term.

For $\kappa \sim 2.9$ (in the scaling model) and $g_\sigma^B/g_\sigma \sim 1.1$ (in the direct coupling model), $B/B_0 \simeq 35 - 40\%$ at the saturation density [8]. It is rewarding that such a significant reduction of the bag constant in the nuclear matter relative to its free-space value *coincides* with that required to recover the relativistic nuclear phenomenology and account for the EMC

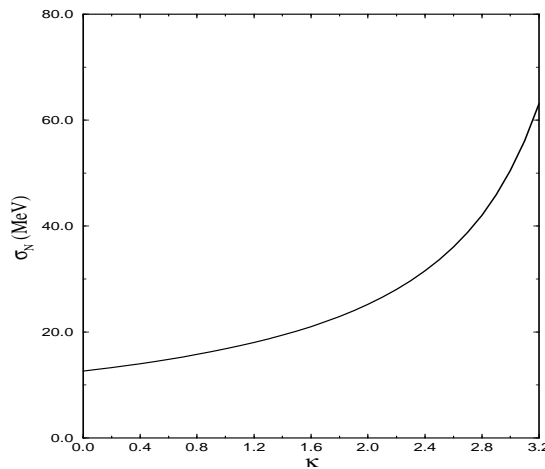


FIG. 1. Nucleon σ term as a function of κ resulting from the scaling model. The case $\kappa = 0$ corresponds to the usual QMC.

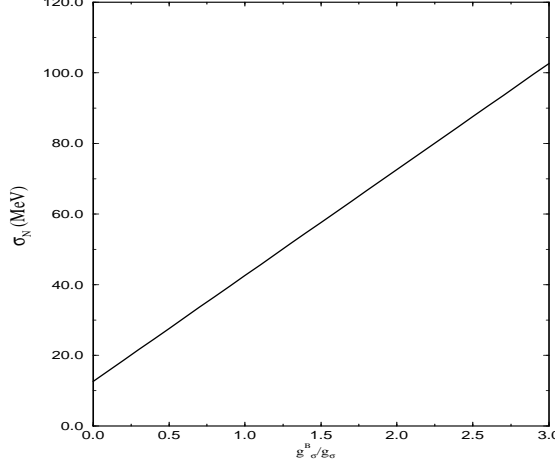


FIG. 2. Nucleon σ term as a function of g_σ^B/g_σ resulting from the direct coupling model. The case $g_\sigma^B/g_\sigma = 0$ corresponds to the usual QMC.

effect [7,8,12]. Conversely, the empirical value $\sigma_N = 45$ MeV provides an extra constraint to the MQMC. In fact, there will be no free parameter in the scaling model and only δ is left as a free parameter in the direct coupling model when σ_N is treated as an input. Then the resulting scalar and vector potentials for the nucleon in the nuclear matter will be comparable to those suggested by the relativistic nuclear phenomenology and finite-density QCD sum rules, and the predictions for the rescaling parameter will be consistent with that required to explain the EMC effect within the dynamical rescaling approach.

We now turn to the in-medium quark condensate. Following Ref. [19], we can write the in-medium quark condensate as

$$\langle \bar{q}q \rangle_{\rho_N} = \langle \bar{q}q \rangle_0 + \frac{1}{2} \frac{d\mathcal{E}}{dm_q} = \langle \bar{q}q \rangle_0 + \frac{1}{2} \left(\frac{\partial \mathcal{E}}{\partial M_N^*} \frac{dM_N^*}{dm_q} + \chi_\sigma \frac{\sigma_N}{m_q} \frac{\partial \mathcal{E}}{\partial m_\sigma} + \chi_\omega \frac{\sigma_N}{m_q} \frac{\partial \mathcal{E}}{\partial m_\omega} \right) \quad (7)$$

where \mathcal{E} is the energy density of the nuclear medium, $\chi_\sigma \sigma_N/m_q \equiv dm_\sigma/dm_q$, $\chi_\omega \sigma_N/m_q \equiv dm_\omega/dm_q$, and $\langle \bar{q}q \rangle_{\rho_N}$ and $\langle \bar{q}q \rangle_0$ denote the quark condensates in the nuclear medium and vacuum, respectively. Here we have followed Ref. [19] and neglected the dependence of various couplings on m_q . Using the Gell-Mann–Oakes–Renner relation, $2m_q \langle \bar{q}q \rangle_0 = -m_\pi^2 f_\pi^2$, one finds [3]

$$R_\rho \equiv \frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_N \rho_N}{m_\pi^2 f_\pi^2} \left[\frac{m_\sigma^2 \bar{\sigma}}{g_\sigma C(0) \rho_N} + \chi_\sigma \frac{m_\sigma \bar{\sigma}^2}{\rho_N} - \chi_\omega \frac{g_\omega^2}{m_\omega^3} \rho_N \right], \quad (8)$$

where m_π is the pion mass (138 MeV) and f_π the pion decay constant (93 MeV). Here we have used $dM_N^*/dm_q = 3m_q C(\bar{\sigma})$ [3,4], which can be obtained by following the same discussion leading to Eq. (6).

To be self-consistent, we use the predictions for σ_N from Eq. (6). (In Refs. [3,4], the empirical value of σ_N was used in evaluating the in-medium quark condensate.) The two

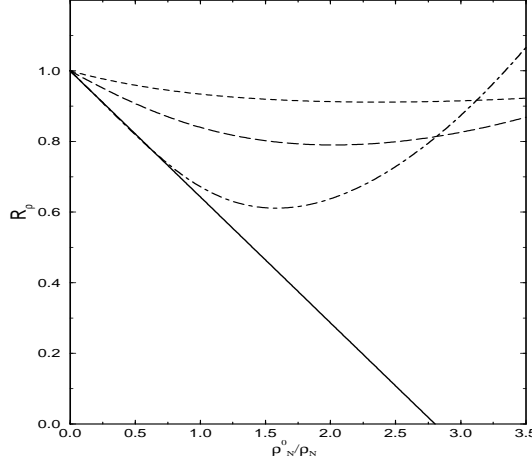


FIG. 3. Ratio $R_\rho = \langle \bar{q}q \rangle_{\rho_N} / \langle \bar{q}q \rangle_0$ as a function of the medium density from the scaling model with $R_0 = 0.6$ fm. The σ_N value is predicted from Eq. (6). The solid curve represents the linear order result with $\sigma_N = 45$ MeV [19], and the other three curves correspond to $\kappa = 0$ (dashed) (usual QMC, $\sigma_N \simeq 12.6$ MeV), 2.0 (long-dashed) ($\sigma_N \simeq 25.2$ MeV), and 2.88 (dot-dashed) ($\sigma_N \simeq 45.0$ MeV), respectively.

coupling constants, (g_σ, g_ω) for the scaling model and (g_σ^B, g_ω) for the direct coupling model, are chosen to fit the nuclear matter binding energy (-16 MeV) at the saturation density ($\rho_N^0 = 0.17 \text{ fm}^{-3}$). Here we also adopt the ansatz $\chi_\sigma \simeq m_\sigma / M_N$ and $\chi_\omega \simeq m_\omega / M_N$ suggested in Ref. [19] with $m_\sigma = 550$ MeV and $m_\omega = 783$ MeV. The predictions from the scaling model are presented in Fig. 3, where R_ρ is plotted as a function of the nuclear medium density for different κ values, with $R_0 = 0.6$ fm. For comparison, the model independent linear order result with $\sigma_N = 45$ MeV, obtained by using $\mathcal{E} = \rho_N M_N$ ($R_\rho = 1 - \sigma_N \rho_N / m_\pi^2 f_\pi^2$) [19], is also shown. For κ values which we are interested here, R_ρ decreases with density at low densities and then starts to increase at higher densities. We see that the dependence of R_ρ on ρ_N is almost linear at low densities with the slope essentially determined by the σ_N value. When $\kappa \simeq 2.88$ ($\sigma_N \simeq 45$ MeV), the result in the region $\rho_N < 1.2 \rho_N^0$ agrees well with the linear order result and starts to grow quickly at $\rho_N \sim 1.8 \rho_N^0$. For smaller (larger) κ values, the result at low densities becomes larger (smaller) than the linear order result, which can be attributed to smaller (larger) σ_N values. The rate of the increase of R_ρ with density at higher densities increases with increasing κ value.

The resulting R_ρ from the direct coupling model is plotted in Fig. 4. The results are for $\delta = 4$ and different values of g_σ^q . For $g_\sigma^q \simeq 1.93$ ($\sigma_N \simeq 45$ MeV), the prediction for R_ρ is in good agreement with the linear order result below $1.2 \rho_N^0$; at $\rho_N \sim 1.9 \rho_N^0$, R_ρ starts to increase with density. As g_σ^q gets larger (smaller), the σ_N becomes smaller (larger) and hence R_ρ becomes larger (smaller). At higher densities, R_ρ increases with increasing density with a large (small) rate for small (large) g_σ^q values. We also tested the sensitivity to the δ value. For a given g_σ^q , the result is not sensitive to δ value in the regime $\rho_N < 2.5 \rho_N^0$, and saturates at $\delta \sim 12$.

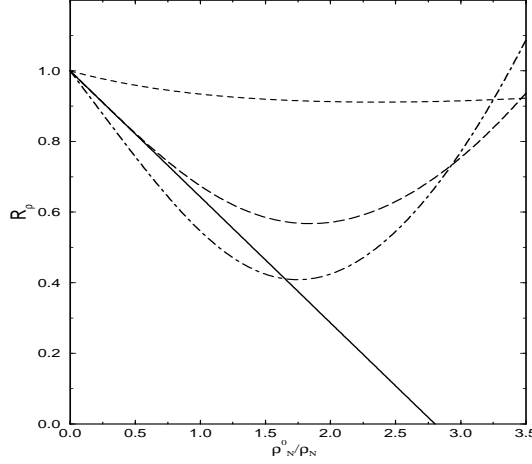


FIG. 4. Ratio $R_\rho = \langle \bar{q}q \rangle_{\rho_N} / \langle \bar{q}q \rangle_0$ as a function of the medium density from the direct coupling model with $\delta = 4$ and $R_0 = 0.6$ fm. The σ_N value is predicted from Eq. (6). The solid curve represents the linear order result with $\sigma_N = 45$ MeV [19], and the other three curves correspond to $g_\sigma^q = 5.309$ (dashed) (usual QMC, $\sigma_N \simeq 12.6$ MeV), 1.93 (long-dashed) ($\sigma_N \simeq 45.0$ MeV), and 1.5 (dot-dashed) ($\sigma_N \simeq 61.2$ MeV), respectively.

We observe that the ratio R_ρ at low densities ($\rho_N \leq \rho_N^0$) is essentially determined by the σ_N value and the linear order result is robust. This is consistent with that found in the usual QMC [3,4] and in hadronic models [19,21–24]. At higher densities, the nonlinear higher-order contributions become increasingly important. In particular, the last term in large parentheses in Eq. (8) becomes dominant, leading to a hindrance of chiral symmetry restoration. This behavior is also seen in hadronic models [21–24]. An exception [23,24] is the ZM model [25], which features density-dependent meson-nucleon coupling constants. However, since the parameters in the MQMC and in hadronic models are chosen to fit only the nuclear matter properties at the saturation density, the reliability of their predictions for the in-medium quark condensate at high densities is unknown. Moreover, the high-density behavior of R_ρ is sensitive to χ_σ and χ_ω , which are not well determined [22]. The possible dependence of the couplings, g_σ^q , g_σ^B , and g_ω , on m_q is also neglected, which would give extra contributions to R_ρ .

In summary, we have evaluated the nucleon σ term and in-medium chiral quark condensate in the modified quark-meson coupling model. The coupling of the bag constant to the scalar mean field featured in the MQMC gives rise to additional contribution to the nucleon σ term compared to the usual QMC. This contribution can lead to the recovery of the empirical value of the nucleon σ term when the reduction of the bag constant in the nuclear matter relative to its free space value is significant, i.e., $B/B_0 \simeq 35 - 40\%$ at $\rho_N = \rho_N^0$. Such a large reduction of the bag constant in the nuclear matter is consistent with that required to recover large and canceling Lorentz scalar and vector potentials for the nucleon in the nuclear matter and to account for the EMC effect within the dynamical rescaling framework.

The resulting in-medium quark condensate at low densities agrees well with the model independent linear order result; at higher densities, the magnitude of the in-medium quark condensate tends to increase, indicating no tendency toward chiral symmetry restoration.

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